MODIFIED METAHEURISTIC ALGORITHMS FOR SINGLE AND MULTI OBJECTIVE SOLID TRANSPORTATION PROBLEMS

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1 Introduction

During WWII, a research group formed under Prof. Blackett proposed strategies to the Allied Forces for optimal use of resources available for war operations. After the war, the strategies used were made public that led to formation of similar groups in industry and later in academia to encourage research with the objective of improving the performance of systems. Hence, the optimization theory or the operations research was formally inducted in the academics. With the advent of computing machines, the research in theory and applications of optimization has grown quite considerably and evident in the varied fields such as Business Economics, Administration, Engineering Design, Manufacturing System, Telecommunication and Town Planning etc.

1.1 Optimization

"Optimization is a procedure of finding and comparing feasible solutions until no better solution can be found" [1]. The general optimization model with m objectives can be organized as:

$$\begin{aligned}
&\text{Minimize/Maximize} && f_m(\mathbf{x}) && m = 1, 2, \ldots, M && \text{(1a)}\\
&\text{subject to} && g_j(\mathbf{x}) \geq 0, && j = 1, 2, \ldots, J && \text{(1b)}\\
& && h_k(\mathbf{x}) = 0, && k = 1, 2, \ldots, K && \text{(1c)}\\
& && {x_i}^{(L)} \leq x_i \leq {x_i}^{(U)} && i = 1, 2, \ldots, n && \text{(1d)}
\end{aligned}$$

where

$\mathbf{x}$	is $(x_1, x_2, \ldots, x_n)^T$, a n-tuple vector of decision variables $x_1, x_2, \ldots, x_n$.
n	is the number of *decision variables*.
M	is the number of objective functions.
$f_m(\mathbf{x})$	is an *objective function* which is to be optimized
J and K	are the number of inequality and equality constraints respectively.
${x_i}^{(L)}$, ${x_i}^{(U)}$	are *lower and upper bounds* of the decision variable x_i.
(1b), (1c)	are *inequality* and *equality constraints* respectively.
(1d)	is *variable bound.*

The model reduces to a single objective optimization problem by considering $m = 1$. The target is to obtain a point $\mathbf{x}$ that satisfies all the constraints and bounds and yields the minimum or maximum value of the objective function.

In case of multiple objectives, there may not exist a single optimal solution that simultaneously optimizes all the objectives as the objectives may be conflicting in nature. In such cases, the trade-off solutions between the respective objectives are considered that do not establish the superiority of one solution over the other. Such solutions, proposed by Vilfredo Pareto, are called Pareto optimal solutions or non-dominated solutions or the efficient solutions.

The Pareto optimal solutions are mathematically defined (for minimization type problem) as: A vector of decision variables $x^* \in S$ for a $k-$objective problem is said to be non-dominated if there does not exist another $x \in S$ such that $f_i(x) \leq f_i(x^*)$ for all $i = 1, 2, \cdots, k$ and $f_j(x) < f_j(x^*)$ for atleast one j.

In other words, a feasible vector x^* is called a Pareto optimal if there is no other feasible solution x that would reduce some objective function without causing a simultaneous increase in atleast one other objective function. The decision maker is free to choose the optimal solution citing his/her priority to the objective functions.

The optimization problems are categorized as linear program if all the objective function(s) as well as the constraint function(s) of the single or multi objective optimization problem are linear otherwise it is a non linear program.

1.2 Transportation Problem

From the point of view of its applications, the transportation problem (TP) is modeled and solved as a special case of minimum cost network flow problem (MCNFP) where the available supply is shipped through the network to satisfy the demand at minimal cost [2]. MCNFP arises in logistics network where people/materials are being moved between various points. MCNFP is applied in design & analysis of communication systems, supply chain and distribution problems, oil pipeline systems, tanker scheduling problems, and a variety of other areas.

Mathematically, TP can be modeled as a discrete Linear Programming Problem (LPP). More specifically, the additional constraints of non-negative integers formulates it as one of the very important Integer Linear Programming Problems (ILP). Over the period, different variants of TP have been introduced for better use of the transportation model. These variants differ from each other on the basis of objective function(s) which are four types, viz., linear & nonlinear TP and single objective & multi objective TP. The objective of TP is, generally, to minimize the cost or time or a trade-off between the two for transportation of a commodity from a number of source points to several destination points. Some of the important variants of TP are discussed hereon.

1.2.1 Classical Transportation Problem

The TP was first presented by Hitchcock [3], along with a constructive solution and, later independently, by Koopmans [4]. Therefore, the classical transportation problem is often referred as Hitchcock-Koopmans' transportation problem. The classical transportation problem **P** with m sources, n destinations is modeled as below:

$$\mathbf{P}: \text{Minimize} \quad \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \tag{2a}$$

$$\text{subject to} \quad \sum_{i=1}^{m} x_{ij} = A_j, \qquad j = 1,2,\ldots,n \tag{2b}$$

$$\sum_{j=1}^{n} x_{ij} = B_i, \qquad i = 1,2,\ldots,m \tag{2c}$$

$$x_{ij} \geq 0, \qquad i = 1,2,\ldots,m,\ j = 1,2,\ldots,n \tag{2d}$$

where

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c_{ij}	is per unit cost of transporting a commodity from i^{th} source to j^{th} destination.
A_j	is the total demand of a commodity at j^{th} destination.
B_i	is the total supply of a commodity available at i^{th} source.
x_{ij}	is the amount of a commodity transported from i^{th} source to j^{th} destination.
(2a)	is objective function to minimize the total cost of the transportation.
(2b), (2c)	are n demand and m supply constraints respectively.
(2d)	are $(m \times n)$ feasibility constraints.

The problem may be stated as to minimize the total cost of transporting a commodity from m sources to n destinations while satisfying all $(m + n)$ constraints of supplies and demands at all sources and destinations along with $(m \times n)$ feasibility constraints. The problem P possesses a feasible solution if and only if it is a *balanced* TP, i.e. the total demand at all destinations is equal to the total supply available at all sources which is mathematically expressed as

$$\sum_{j=1}^{n} A_j = \sum_{i=1}^{m} B_i \tag{3}$$

1.2.2 Time Minimization Transportation Problem

In the problems like transporting food, ammunition, medicines etc for the soldiers in war-like situations, shipping of perishable items such as life saving drugs, fresh fruits and vegetables etc, transportation time of a commodity plays very important role. This motivated Hammer [5] for formulating the Time Minimization Transportation Problem (TMTP). Garfinkel and Rao [6], Szwarc [7] and other researchers have contributed to TMTP.

The mathematical model of TMTP defines the transportation time in a source-destination pair $(i - j)$ as t_{ij} which is independent of the quantity being transported between the pair. The TMTP carries the basic structure of the classical model with the exception of objection function. It is modeled as under:

$$\textbf{Q: Minimize} \quad T(X) = \max_{\substack{1 \le i \le m \\ 1 \le j \le n}} \{t_{ij} : x_{ij} > 0\} \tag{4}$$

subject to the Constraints **(2b)**, **(2c)**, **(2d)**, defined in Section **1.2.1**.

It is assumed that, **(i)** the carriers have sufficient capacity to carry goods from a source to a destination in a single trip, **(ii)** transportation starts simultaneously from sources and **(iii)** the problem is a balanced one. The cost-time trade-off, to minimize both the cost and time of transportation, too has been of great interest.

Researchers have been attracted to some other variants of TP such as Bulk Transportation Problem (BTP), Solid Transportation Problem (STP), Fixed-Charge Transportation Problem and even extended these to multi-objective ones. The BTP, formulated by Maio and Roveda [8], stipulates that the demand of a destination is met from a single source only even through a source may supply to any number of destinations subject to the availability of the commodity at the source. This suggests that

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the model is similar to Generalized Assignment Problem wherein a job is assigned to a unique worker though he/she may be assigned more than one job depending on his/her availability. The primary objective in both these problems is to minimize the total cost. The other models pertaining to the objectives of the proposed work are presented ahead.

1.3 Solution Techniques

The solution of the ILP is based on the idea of selection of a point (solution) from a finite set of possible (candidate) solutions. Such solutions consist of the collection of vertices of convex polytope formed by the linear constraints in the problem. This solution determining method categorizes ILP, and, in particular TP, as one of the combinatorial optimization problems.

Mathematical structure of TP suggests that it is amenable to solution by simplex method, though better techniques such as Modified Distribution (MODI) [9] and Stepping Stone Method [10] are used to solve the same owing to the nature of the problem and requirements of simplex method. These methods, too, operate in the same manner as starting from an Initial Basic Feasible Solution (IBFS) generally obtained through Vogel's Approximation, Extremum Difference, Russel's Approximation Method, etc and iterating towards optimal solution. However, these techniques fail to provide a solution in polynomial time as the problems become NP-Hard with the increase in dimension or the complexity of the variants of the TP [11, 12].

Solving these problems need powerful techniques and with the suitable mathematical programming, the decision makers can certainly save huge amount of effort and time. This premise led to the development of heuristic or meta-heuristic techniques that gained importance because of their low computational complexity, easy implementation and relatively few parameters. Many heuristic and metaheuristic algorithms are available in literature to solve TP and its variants [13, 14, 15, 16].

2 Problem Formulation

The four variants of Solid Transportation Problem (STP), based on the types of objective function(s), are identified for this study. In the literature, STP is also named as Multi-Index Transportation Problem or Three-Index Transportation Problem. Therefore, hereafter the names Solid and Multi-Index are used interchangeably. The mathematical models of the problems are formulated hereon.

2.1 Solid Transportation Problem

Haley [17, 18] introduced solid transportation troblem (STP) where p types of commodities are transported from each of m sources to each of n destinations. The problem is generalization of classical TP and it reduces to one when p is assumed equal to 1. The decision variable x_{ijk} deals with p types of commodities through the index k while the indices i and j have usual meaning. The problem may be extended to *four-index* by considering an additional parameter, say mode of transportation or type of vehicle, etc. Junginger [19] formulated the generalized n-index transportation problem and

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explored various possibilities of formulation of the problem. The mathematical model of the problem is given ahead.

2.1.1 Single Objective Solid Transportation Problem

The mathematical model of this STP comprising of m sources, n destinations and p types of commodities with the single objective to minimize the total transportation cost is written as:

$$\mathbf{P_1}: \text{Minimize} \quad \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} c_{ijk}x_{ijk} \tag{5a}$$

subject to

$$\sum_{i=1}^{m} x_{ijk} = A_{jk} \qquad j = 1,2,\ldots,n;\ k = 1,2,\ldots,p \tag{5b}$$

$$\sum_{j=1}^{n} x_{ijk} = B_{ki} \qquad i = 1,2,\ldots,m;\ k = 1,2,\ldots,p \tag{5c}$$

$$\sum_{k=1}^{p} x_{ijk} = E_{ij} \qquad i = 1,2,\ldots,m;\ j = 1,2,\ldots,n \tag{5d}$$

$$x_{ij} \geq 0 \quad i = 1,2,\ldots,m;\ j = 1,2,\ldots,n;\ k = 1,2,\ldots,p \tag{5e}$$

where

c_{ijk}	is per unit cost of transporting k^{th} commodity from i^{th} source to j^{th} destination.
A_{jk}	is the total amount of k^{th} commodity transported at j^{th} destination.
B_{ki}	is the total amount of k^{th} commodity available at i^{th} source.
E_{ij}	is the amount of all p types of commodities transported from ith source to j^{th} destination.
x_{ijk}	is the amount of k^{th} commodity transported from i^{th} source to j^{th} destination.
(5a)	is the objective function of minimization of total cost of transportation of p types of commodities from m sources to n destinations.
(5b)	the sum of supplies from all sources to j^{th} destination of k^{th} commodity
(5c)	the sum of supplies from i^{th} source to all destinations of k^{th} commodity
(5d)	sum of supplies from i^{th} source to j^{th} destination of all commodities
(5e)	are $m \times n \times p$ feasibility constraints.

The problem P_1 is said be balanced if it satisfies the following single and double sum equations:

$$\begin{gathered}\sum_{j=1}^{n} A_{jk} = \sum_{i=1}^{m} B_{ki};\ \sum_{k=1}^{p} B_{ki} = \sum_{j=1}^{n} E_{ij};\ \sum_{i=1}^{m} E_{ij} = \sum_{k=1}^{p} A_{jk} \\ \sum_{j=1}^{n}\sum_{k=1}^{p} A_{jk} = \sum_{k=1}^{p}\sum_{i=1}^{m} B_{ki} = \sum_{i=1}^{m}\sum_{j=1}^{n} E_{ij}\end{gathered} \tag{6}$$

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The tabular representation of the problem is given below.

Table 1: Tabular representation of Solid Transportation Problem

Sources	Destinations									Capacity
	D_1			D_2			D_j	D_n		
S_1	c_{111}	c_{112}	E_{11} c_{113}	c_{121}	c_{122}	E_{12} c_{123}	.. ..	c_{1n1}	c_{1n2} E_{1n} c_{1n3}	B_{11} B_{21} B_{31}
S_2	c_{211}	c_{212}	E_{21} c_{213}	c_{221}	c_{222}	E_{22} c_{223}	.. ..	c_{2n1}	c_{2n2} E_{2n} c_{2n3}	B_{12} B_{22} B_{32}
S_i	..	..	.. ..	..	..	.. ..	.. ..	..	.. ..	.. .. ..
S_m	c_{m11}	c_{m12}	E_{m1} c_{m13}	c_{m21}	c_{m22}	E_{m2} c_{m23}	.. ..	c_{mn1}	c_{mn2} E_{1n} c_{mn3}	B_{1m} B_{2m} B_{3m}
Demand	A_{11}	A_{12}	A_{13}	A_{11}	A_{12}	A_{13}	.. .. ..	A_{11}	A_{12} A_{13}	

The problem possesses a feasible solution provided it satisfies all the constraints and the feasibility conditions. There may exist more than one solution, with different values of the decision variable, with the optimal value of the objective function. Such solutions are called the alternate optimal solutions.

2.1.2 Multi Objective Solid Transportation Problem

As discussed earlier that the time of transportation also plays significant role in decision making, hence it is quite natural to discuss the MITP with multiple objectives to minimize the cost and time simultaneously. The Multi-Objective Solid Transportation Problem (MOSTP) is formulated as:

$$\mathbf{P_2}: \text{Minimize} \left\{ \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} c_{ijk}x_{ijk}, \max_{\substack{1\le i\le m\\ 1\le j\le n\\ 1\le k\le p}} \{t_{ijk};\ x_{ijk}>0\} \right\} \tag{7}$$

subject to the constraints **(5b)** to **(5e)** of Section 2.1.1.

where

t_{ijk}	is the time of transporting k^{th} type of commodity from i^{th} source to j^{th} destination.
$\max_{i,j,k}\{t_{ijk};\ x_{ijk}>0\}$	is the maximum time corresponding to the non-zero allocations x_{ijk}

The time t_{ijk} is independent of the quantity transported of any type of commodity transported between any source-destination pair. The objectives of the problem are conflicting in nature as improvement in one compromises the optimality of the other, thereby leading to the identification of Pareto optimal solutions.

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2.2 Fixed-Charge Transportation Problem

The real life transportation involves some fixed expenses such as rent of vehicle, tolls on a route, freight rates, landing fee at airport or some set costs, etc. These costs have a direct impact on transportation planning so rather than just considering the variable cost of transportation the fixed-charges are also considered to get an optimal transportation decision. Hirsch and Dantzig [20] initialized the Fixed-Charge Transportation Problem (FCTP) problem by extending the classical TP. The FCTP reduces to classical TP as the fixed costs are assumed equal to zero. The problem has been extended for transporting multiple commodities from sources to destinations and the mathematical model is produced in the next subsection.

2.2.1 Single Objective Fixed Charge Solid Transportation Problem

The problem of fixed-charges being incurred alongwith the variable costs of transporting heterogeneous commodities from various sources to many destinations is being considered with the objective of minimizing the total transportation cost (variable and fixed). This problem is known as Fixed-Charge Solid Transportation Problem (FCSTP). The cost of transportation, in this case, is not directly proportional to the quantity of commodity transported. The objective function of MFCTP is to minimize the sum of total cost of transportation and fixed charges incurred due to transportation of any type of commodity from any source to any destination. The mathematical model of the problem is as follows:

$$\mathbf{P_3}: \text{Minimize} \left\{ \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} c_{ijk}x_{ijk} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} F_{ijk}Y_{ijk} \right\} \tag{8a}$$

subject to the constraints **(5b)** to **(5e) of Section 2.1.1.**

$$\text{where} \qquad Y_{ijk} = \begin{cases} 1 & if\ x_{ijk} > 0 \\ 0 & if\ x_{ijk} = 0 \end{cases} \tag{8b}$$

Y_{ijk} ensures that the fixed charge F_{ijk} is added to the total cost of the transportation only if some non-zero amount of commodity of type k is transported from i^{th} source to j^{th} destination. The objective (8a) is to minimize the sum of total variable cost and total fixed cost to yield an optimal basic feasible solution subject to the mentioned constraints. The problem P_3 is also assumed to satisfy feasibility conditions (6).

2.2.2 Multi-Objective Fixed-Charge Solid Transportation Problem

The consideration of time of transportation has been studied critically in some variants of TP. Hence, when contemplated with the costs (variable and fixed) in case of the transportation of heterogeneous commodities, FCSTP can be extended to Multi-Objective Fixed Charges Solid Transportation Problem (MOFCSTP) wherein the problem becomes a concave optimization problem to be studied over a convex solution

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space. This problem is mathematically modeled as:

$$\mathbf{P_4}: \text{Minimize} \left\{ \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} c_{ijk}x_{ijk} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} F_{ijk}Y_{ijk}, \right.$$
$$\left. \max_{\substack{1\le i\le m \\ 1\le j\le n \\ 1\le k\le p}} \{t_{ijk};\ x_{ijk} > 0\} \right\}$$

subject to the constraints **(5b)** to **(5e)** of Section 2.1.1 and **(8b)** of Section 2.2.1. (9)

The problem P_4 is also assumed to satisfy feasibility conditions (6).

3 Literature Review

3.1 Metaheuristic Solution Techniques

The word "metaheuristic" was coined by Fred Glover [21]. "A metaheuristic is formally defined as an iterative generation process which guides a subordinate heuristic by intelligently combining different concepts for exploring and exploiting the search space, learning strategies are used to structure information in order to find efficiently near-optimal solutions." [22]. The exploration is responsible for searching best solution in the surrounded space while the exploitation explores new possible search areas which remained hidden during exploration due to some structural problem of the heuristic algorithm.

Metaheuristic algorithms are paradigms of computational intelligence which are particularly used to solve sophisticated and complex optimization problems [23]. These are recognized and differentiated based on their searching approach as well as the computational effort carried out to attain the optimal solution. For complex optimisation problems, metaheuristic techniques are able to produce good quality solution in comparatively much less time than conventional optimisation techniques [24]. Metaheuristics have been implemented on numerical problems which include continuous and discrete, constrained and unconstrained, single and multi-objective optimization problems, etc [25] for telecommunications, engineering, machine learning, logistics and planning.

The emphasis is on the best possible solution for the problem, not the promotion of a certain metaheuristic [26]. The metaheuristics are selected due to their efficient performance, high number of citations, specific evolutionary operators, interesting interaction mechanisms between individuals, parameter tuning/handling concepts, and stagnation prevention methods [27]. Some of these techniques to be considered to application to the objectives of the work are presented ahead.

3.1.1 Genetic Algorithm

Genetic Algorithms (GAs) are the search algorithms based on mechanics of natural selection and natural genetics. They require the natural parameter set of the optimization problem to be coded as a finite-length string over some finite alphabet. GA

starts with a population of strings and thereafter generates successive populations of strings through the three operators, viz., (i) Reproduction, (ii) Crossover and (iii) Mutation. Reproduction is a process in which individual strings are copied according to their objective function values. Crossover operator after reproduction the characters of each pair of strings are swapped while mutation is the occasion (with small probability) random alteration of the value of a string position.

Holland [28] introduced the GA followed by Goldberg [29] who presented the first original form of the GA. Further, GAs have been extensively studied by Goldberg [30, 31], Michalewicz et al.[15, 32], Gen and Cheng [33], Syariff [34], Gottlieb [35] in wide variety of applications and forms. Recently, Volodymyr Skitsko and Mykola Voinikov[36] have solved the four-index transportation problem with GA.

3.1.2 Particle Swarm Optimization

Particle swarm optimization (PSO), originally designed and introduced by Eberhart and Kennedy [37], is a population based search algorithm based on the simulation of the social behaviour of birds, bees or school of fish. The algorithm simulates the graceful and unpredictable choreography of a bird flock. Each individual within the swarm, represented by a position vector, is assigned velocity through which it moves to its next improved position that is guided by its own and swarm's best positions visited.

Assuming the search space to be n-dimensional, the i^{th} particle of the swarm is represented by $X_i = (X_{i1}, x_{i2}, \ldots, x_{in})$ and the velocity by $V_i = (v_{i1}, v_{i2}, \ldots, v_{in})$. The best visited position of X_i and global best position of the swarm may be considered as $P_{i,best} = (p_{i1}, p_{i2}, \ldots, p_{in})$ and $P_{g,best} = (p_{g1}, p_{g2}, \ldots, p_{gn})$. The velocity and the position of the particle is updated using the following equations:

$$V_i(t+1) = \omega V_i(t) + c_1\phi_1(P_{i,best} - X_i) + c_2\phi_2(P_{g,best} - X_i) \tag{10a}$$

$$x_i(t+1) = x_i(t) + v_i(t+1) \tag{10b}$$

where ω is the inertia that contributes the effect of the previous velocity of the particle; c_1 and c_2 are positive constants that denote the cognitive and social acceleration coefficients respectively; ϕ_1 and ϕ_2 are two random variables with uniform distribution between 0 and 1.

PSO is easier to implement since it has fewer parameters to adjust and is more efficient is maintaining the diversity of the swarm, since all the particles use the information related to the most successful particle in order to improve themselves. PSO is a computationally intelligent technique that is not largely affected by the size and nonlinearity of the problem, and can converge to the optimal solution in many problems where most analytical methods fail to converge. PSO has been used in wide areas of application such as electrical and electronic engineering, automation control systems, communication theory, operations research, mechanical engineering, fuel and energy, medicine, chemistry, and biology [38]. In recent times, Singh and Singh have proposed a Hybrid-PSO to solve balanced TP [39] and modified PSO to solve Time Minimization TP [40].

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3.1.3 Simulated Annealing

Kirkpatrick, Gelett and Vecchi [41] proposed a probabilistic method, called Simulated Annealing (SA), for finding a global minimum of a multi-modal function, i.e. a function with several local minima. The name "simulated annealing" is given from the analogy with the physical annealing process of solids. SA consists of a sequence of moves from an incumbent solution to an improved one according to certain transition rules while accepting occasionally some uphill solutions in order to guarantee a diversity in the domain exploration and to avoid getting caught at local optima. It is possible to give a polynomial upper bound on computational time for some implementations of the algorithm [42]. The other names that have been used to denote these algorithms are Probabilistic hill climbing [43], statistical cooling [44], and stochastic relaxation [45]. SA-based algorithms solve single and multi-objective optimization problems of discrete as well as continuous type the summary of some important approaches is presented in the survey [46]. A hybridized adaptive tabu-simulated annealing algorithm is developed for concave transportation problem by Altiparmak [47].

3.1.4 Differential Evolution

Storn and Price [48] proposed Differential Evolution (DE) algorithm, a simple yet powerful population-based stochastic search technique that provides an efficient and effective global optimizer in the search field. There are several trial vector generation strategies in DE, some of which may be sufficient to address a particular problem. The DE's optimization performance can be significantly influenced by three main control parameters, viz., population size, scaling factor and crossover rate. DE has found applications in wide variety of areas as mentioned in [49, 50, 51, 52].

3.1.5 Ant Colony Optimization

The Ant Colony Optimization (ACO) was initially proposed by Dorigo and coworkers [53, 54, 55] and was inspired by the food-foraging behavior of ant colonies. Ants are social insects, that is, insects living in colonies, whose behavior focusses more on the survival of the whole colony than on the survival of one particular individual member of the colony. The scientific community has operated extensively to expand and apply anti-based algorithms to a wide variety of continuous [56, 57] as well as discrete optimization problems [58, 59, 60].

3.2 Solid Transportation Problem

The solid transportation problem was first stated by Schell [61], then Haley [18] generalized the MODI (Modified Distribution) method to solve multi-index transportation problem. Generalized model for n-index (n is positive integer) transportation problem was introduced by Junginger [19] where possibilities for different variants of n-index TP were discussed. A technique for solving time minimization solid transportation problem was discussed by Bhatia and Swarup [62]. Later, Bhatia and Puri [63] proposed a time-enumerative technique for obtaining successive time-cost trade-off solutions in a bi-criteria solid transportation problem. Zen, Ida, Li [64] then studied the bi-criteria solid transportation problem by Genertic Algorithm. Later, Ahuja & Arora

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[65] and Khurana & Adlakha [66] proposed algorithms to find out all possible efficient cost–time trade-off pairs in the multi-index fixed charge bi-criterion transportation problem. Khurana and Adlakha [67] then addressed multi-index constrained TP with axial constraints with bounds on availability, requirement and commodities.

4 Research Gaps

As mentioned earlier, the complexity of the TP increases beyond polynomial time due to the size &/or complex structure of the problem and the optimality test of IBFS therein. Therefore, the traditional methods become impractical to imply and the focus shifts to heuristic or metaheuristic techniques. However, the specific need of solid transportation model, that is, the decision variables to be non-negative integers, is generally not met by these existing heuristic methods. Hence, the techniques need to be modified as per the requirement of the problem at hand. This has generally been done by rounding off techniques as discussed by Jordehi and Jasni[68]. In this approach, discrete/integer variables are treated the same with continuous variables during the course of optimization but at the end of process, they are rounded off to the nearest discrete/integer value. This procedure produces a set of feasible values of variables that improves the value objective function but may not satisfy the constraints of the problem. This implies that there is an urgent need to introduce an algorithm that is amalgamated with the existing (meta)heuristic technique so as to obtain a solution that not only satisfies the constraints but also the feasibility conditions while optimizing the objective function(s).

5 Contribution and Need of the Proposed Research

With the issues arising out of the pairs of optimization problem - solution techniques in mind, it is realised that the solution techniques need to be moderated to suit the requirements of the respective problem, be it the case of objective function, the constraints or the feasibility conditions. Thus study is being proposed with an intention to contribute modified and/or hybridized (meta)heuristic algorithms to find optimal solutions, near optimal solutions or Pareto-optimal solutions, as applicable, with least computational effort and exhaustive search capability for solving the solid transportation problem and its variants. The proposed algorithms shall be designed with a view to improve the quality of the solutions, rate of convergence and better search capability. This work will study and discuss some strategies related to PSO implementation such as parameter setting, convergence behaviour & stability analysis, check of diversity loss etc. and will also embed problem specific knowledge and identify the areas of extension of the work.

6 Objectives

In the previous sections, the TP has been discussed with some of its variants. These problems have been studied with single &/or multiple objectives and researchers have

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solved them using different analytic and heuristic techniques.
It is proposed to study the following problems and solve them using the metaheuristic algorithms.

1. Single Objective Solid Transportation Problem
2. Multi Objective Solid Transportation Problem
3. Single Objective Fixed Charge Solid Transportation Problem
4. Multi Objective Fixed Charge Solid Transportation Problem

7 Methodology

The objectives of the work shall be taken up for solution through the metaheuristic algorithms with appropriate modification or hybridization of two or more such techniques. Some new algorithms may be developed to amalgamate with the existing (meta)heuristic techniques with an intent to improve the convergence rate of the algorithms or meeting the specific requirements of the combinatorial optimization problems at hand.
The algorithms shall be coded in MATLAB to determine their rate of convergence to the optimal solution or Pareto optimal solution, as the case may be. The statistical validation of the algorithms and the solutions shall be carried out to establish the efficiency and reliability of the solution techniques for the respective problems.

8 Planning of Work

It is planned to organize the work flow in the following manner:

Work	Semesters					
	I	II	III	IV	V	VI
Literature Survey	✓	✓	✓	✓		
Designing and Testing Algorithms		✓	✓	✓	✓	
Publications			✓	✓	✓	✓
Thesis Writing				✓	✓	✓

9 Facilities

The following facilities are required to carry out the proposed work.

1. Laptop with Windows operating system
2. Softwares (MATLAB, Microsoft Office and LaTeX)
3. Printer

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10 Place of Work

The research work shall be carried out at Baba Banda Singh Bahadur Engineering College, Fatehgarh Sahib.

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